Balanced Data and Unbalanced Data of the Strip-Strip-Plot Designs for Three Multi-Stages of Metal Stamping Processes

การออกแบบการทดลองสตริป-สตริป-พล็อตแบบสมมาตรและไม่สมมาตรสำหรับสามกระบวนการต่อเนื่องของกระบวนการปั๊มโลหะ

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ABSTRACT

For a metal plate battery carrier, its stamping production line composes of five stages: blanking, forming, flanging, piercing-1 and piercing-2. The stamping processes generate unexplained inherent variations that result in hole-misalignment in the final stage. In this paper, the purposes are to utilize experimental designs of three multi-stages processes (multi-stages DOE), and to extract significant factors from all stages. Full factorial designs are chosen in which all treatment combinations are under strip-strip-plot structure. Balanced data and unbalanced data designs are generated with the D-optimal efficiencies at 80% and 69.87% respectively and implemented in a real stamping processes. An analysis of multiple regression is used for their generalized model estimators significantly with two cross-stage interaction effects between stage-2 and stage-3. Both designs provide predicted models in which their generalized estimators are slightly different. However, with hypothesis testing using t-test, both predicted models provide no significant difference in their predicted values. Thus, unbalanced data design, with 24 runs, is more efficiently implemented than balanced data design, with 32 runs, in term of less experimental cost.

KEYWORDS: Strip-Strip-Plot Design, Multi-Stage Processes, Cross-Stage Interaction Effects, D-optimal

บทคัดย่อ

กระบวนการปั๊มโลหะสำหรับแผ่นรองแบตเตอร์รี่รถยนต์ประกอบด้วยกระบวนการย่อย 5 กระบวนการ ได้แก่ การตัดแผ่นเปล่า การขึ้นรูป การพับขึ้นรูป การเจาะ-1 และการเจาะ-2 เนื่องจากประกอบด้วยหลายกระบวนการ จึงเกิดการสะสมของความแปรปรวนมากขึ้น จนทำให้ขนาดของรูยึดไม่ได้ตามข้อกำหนดที่กระบวนการสุดท้าย งานวิจัยนี้มีจุดประสงค์ในการออกแบบการทดลองสำหรับสามกระบวนการ เพื่อดึงปัจจัยที่มีความสำคัญอย่างมีนัยสำคัญออกมาจากทุกกระบวนการ การทดลองเชิงแฟคทอเรียลเต็มรูปได้ถูกนำมาใช้ภายใต้โครงสร้างของการออกแบบการทดลองสตริป-สตริป-พล็อต แบบแผนการทดลองได้ถูกออกแบบให้มีทั้งแบบที่โครงสร้างข้อมูลมีความสมมาตร และแบบที่โครงสร้างข้อมูลมีความไม่สมมาตร ประสิทธิภาพของแบบแผนการทดลองถูกประเมินด้วยค่าดี-เหมาะสมที่สุด ซึ่งมีค่าร้อยละ 80 และร้อยละ 69.87 ตามลำดับ และได้นำไปทดลองจริงในกระบวนการปั๊มโลหะ ข้อมูลการทดลองที่ได้มาได้ถูกนำมาวิเคราะห์โดยใช้การวิเคราะห์การถดถอยพหุคูณ ซึ่งสามารถดึงเอาปัจจัยร่วมระหว่างกระบวนการที่มีผลอย่างมีนัยสำคัญออกมาได้ อย่างไรก็ตาม แบบแผนทั้งสองแบบได้ให้ค่าที่เหมาะสมที่สุดแตกต่างกันเล็กน้อย แต่เมื่อทำการทดสอบสมมติฐานด้วยสถิติทดสอบที ความแตกต่างนี้ไม่มีนัยสำคัญ ดังนั้น แบบแผนการทดลองแบบไม่สมมาตร ซึ่งมีจำนวนการทดลองเพียง 24 การทดลอง จึงมีประสิทธิภาพมากกว่าแผนการทดลองแบบสมมาตร ซึ่งมีจำนวนการทดลอง 32 การทดลอง เนื่องจากเสียค่าใช้จ่ายในการทดลองน้อยกว่า

คำสำคัญ: การออกแบบการทดลองสตริป-สตริป-พล็อต การออกแบบการทดลองสำหรับสามกระบวนการ ปัจจัยร่วมระหว่างกระบวนการ ค่าดี-เหมาะสมที่สุด

Introduction

For a plate batter carrier, its stamping production line composes of five processes causing inconsistency in quality in the final process due to accumulated inherit variations. The cross-stage interaction effects are those of unexplained causes that can lead to final product out-of-specification even though all process parameters are under controlled.

To study inherit cross-stage interactions or variations of multi-stage processes, strip-plot design is chosen because of its stripped structure. The three-way, row and column and cell are called strip-strip-plot design and can apply to three multi-stages DOE. However, strip-strip-plot structure generated a large number of experimental runs.

From our previous work, we generated several patterns of second-order model within strip-strip-plot structure with generalized least square and ordinary least square (GLS-OLS) equivalent condition. These patterns can be analyzed with common statistical software but their runs are still not optimized with some redundancies.

Literatures on the application of strip-plot design or strip-strip-plot design has rarely been seen and most of them were in agriculture than in industry. An industrial research on strip-plot design was performed in two processes at a battery factory on the purpose of cost reduction by using half fractional factorial design in stage-2. The first strip-strip-plot design was experimented with three processes in a wafer factory by using half fractional factorial design in the stage-3. D-optimal criterion was introduced into strip-plot designs on the purpose of reducing experimental runs and was utilized in two processes (Arnouts, Goos, & Jones, 2010). and three processes (Arnouts, Goos, & Jones, 2013).

D-optimal strip-strip-plot designs are generated by an algorithm search that involve generalized variance-covariance matrix (Arnouts, Goos, & Jones, 2012). Many recent researches have been worked for similar designs, split-plot design and split-split-plot design, especially on second-order split-plot design searching algorithm (Nguyen & Pham, 2015). The currently interesting researches are the statistical inference of split-plot and multi-stratum designs (Trinca & Gillmour, 2017). An interesting search algorithm can deal with any numbers of stages and provide six optimal criteria (Borrotti, Sambo, Mylona, & Gilmour, 2017).

In last three years, our attempts had worked for D-optimal strip-strip-plot design for second-order model with output of forty-five patterns (Tantiphanwadi & Sudasna na Ayudthya, 2016). Two experiments had been performed in a food factory of pork ham and macaroni ready-meal (Tantiphanwadi & Sudasna na Ayudthya, 2017). A recent one is D-optimal designs based on the second-order least squares estimator (Gao & Zhou, 2017). To extend our research, balanced data and unbalanced data strip-strip-plot designs are performed in stamping production line to increase product quality with minimum experimental costs.

Purposes

The research is aimed to implement three multi-stages DOE in metal stamping production line in which contains five sub-processes or stages with minimum number of experimental runs. The cross-stage interaction effects should be significantly extracted from the experiment and included in predicted model that can provide the optimum parameters to reach the acceptable quality.

Another purpose is aimed to studying both balanced data (OLS-GLS equivalence) and unbalanced data (GLS) designs. The test results of them will be compared for the minimal runs which resulted in less experimental cost.

Benefit of Research

The effectiveness of three multi-stage DOE can be evaluated in metal stamping processes in which its design pattern contain minimum experimental runs. Experimental cost will be more minimized compared with those of single DOE patterns.

Research Process

1. Stamping Processes

The chosen product is plate battery carrier with high nonconformance as hole displacement, measured from the two lengths of left or right edges to the center of the hole. The different specification between left and right displacements should be 0.00±0.50 mm. Even though all process parameters are under control, the inherit variations are still temporary existing.

Stamping production line composes of five processes shown in Figure 1. A roll of metal sheet is pulled into blanking process which it cuts metal sheet into many small pieces of plate battery carrier. Then, each piece will be manually feeded into next processes as forming, flanging, piercing-1, and piercing-2 consecutively.

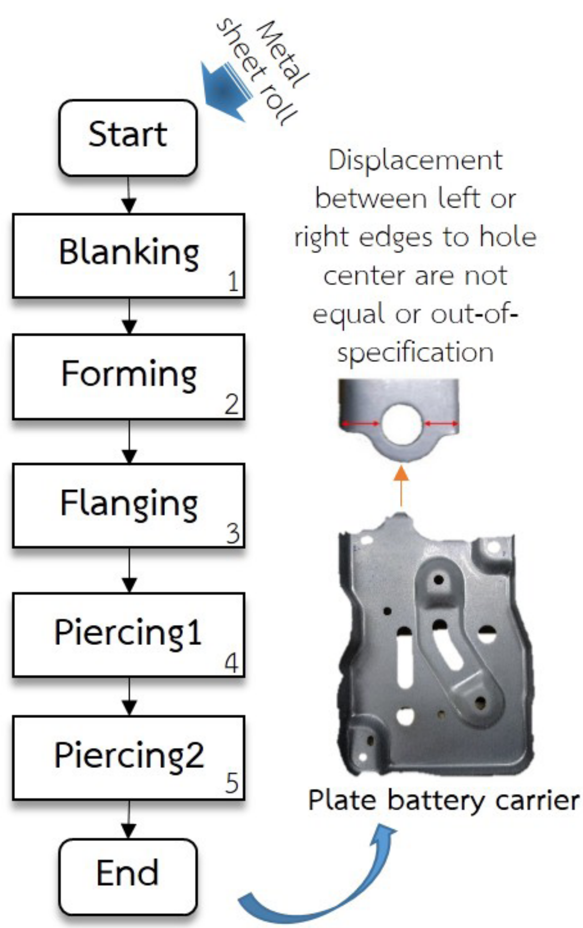


Figure 1 Stamping process with five stages

2. Identifying Generated Inherit Variation Processes

10 plates are randomly selected at blanking process and run from process-2 through process-5. In each process, output critical points are measured. The points that their standard deviations are higher than 0.10 mm are treated as out-of-limit points. In Table 1, process-2, process-3 and process-5 are significantly chosen because of high percentage values of 60%, 100% and 100% respectively.

Table 1 Measured information of critical points

Process/Critical Points	% out-of Limit Points	Max.StDev. (σ, mm)
1: Blanking	50%	0.103
2: Forming	60%	0.185
3: Flanging	100%	0.165
4: Piercing1	50%	0.159
5: Piercing2	60%	0.155

3. Mathematical Models

3.1 Strip-strip-plot structure

In a strip-strip-plot structure, row treatments (T) are crossed with column treatments. Cell treatments occur at each row-column combination. Let r, c, k are applied for the number of runs of row, column and cell respectively. The total runs, $N = \sum_{i=1}^{r} \sum_{j=1}^{c} r_i c_j k$. Thus, stage-1, stage-2 and stage-3 can be represented by row, column and cell respectively. The structure is guaranteed that cross-interaction effects of all stages can be observed. The crossing structure is shown in Table 2.

3.2 Strip-strip-plot model

In a strip-strip-plot structure, the two-way classification random model with balanced data or unbalanced data is utilized (Searle, Casella & McCuloch, 2006).

Table 2 Crossing structure of three multi-stage processes

		Stage-2			
		$\mathbf{T}_1$	$\mathbf{T}_2$	$\cdots$	$\mathbf{T}_c$
		Stage-3	Stage-3	Stage-3	Stage-3
Stage-1	$\mathbf{T}_1$	$\mathbf{T}_{111}$ $\vdots$ $\mathbf{T}_{11k}$	$\mathbf{T}_{121}$ $\vdots$ $\mathbf{T}_{12k}$	$\vdots$	$\mathbf{T}_{1c1}$ $\vdots$ $\mathbf{T}_{1ck}$
	$\mathbf{T}_2$	$\mathbf{T}_{211}$ $\vdots$ $\mathbf{T}_{21k}$	$\mathbf{T}_{221}$ $\vdots$ $\mathbf{T}_{22k}$	$\vdots$	$\mathbf{T}_{2c1}$ $\vdots$ $\mathbf{T}_{2ck}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	$\mathbf{T}_r$	$\mathbf{T}_{r11}$ $\vdots$ $\mathbf{T}_{r1k}$	$\mathbf{T}_{r21}$ $\vdots$ $\mathbf{T}_{r2k}$	$\vdots$	$\mathbf{T}_{rc1}$ $\vdots$ $\mathbf{T}_{rck}$

The three multi-stage processes model is introduced in matrix notation as

$$Y = X\beta + Z_\delta \delta + Z_\gamma \gamma + \varepsilon \qquad (1)$$

where

Y is a $N \times 1$ vector of experimental data,

β is a $p \times 1$ fixed effect parameters,

X is a $N \times p$ coefficient matrix,

Z_δ and Z_γ are row and column incidence matrice,

δ and γ are row and column random effect vectors,

ε is a random effect of cell or $N \times 1$ vector.

It is assumed that random effects, $\delta \sim N(0_r, \sigma_\delta^2 I_r), \gamma \sim N(0_c, \sigma_\gamma^2 I_c)$ and $\varepsilon \sim N(0_N, \sigma_\varepsilon^2 I_N)$, where 0 and I are zero vectors and identity matrice are normally distributed around zero mean vectors. Their variances; σ_δ^2, σ_γ^2 and σ_ε^2 are referred in stage-1, stage-2 and stage-3 respectively.

Another assumption is that their covariance, $\mathrm{cov}(\delta, \gamma) = 0_{r \times c}$, $\mathrm{cov}(\delta, \varepsilon) = 0_{r \times n}$ and $\mathrm{cov}(\gamma, \varepsilon) = 0_{c \times N}$ are zero because each of them is independent to the others. The strip-strip-plot model Y, as in equation (1), contains variance-covariance matrix $V = \mathrm{var}(Y)$ as follows:

$$\mathbf{V} = \boldsymbol{\sigma}_\delta^2 \mathbf{Z}_\delta \mathbf{Z}_\delta' + \boldsymbol{\sigma}_\gamma^2 \mathbf{Z}_\gamma \mathbf{Z}_\gamma' + \boldsymbol{\sigma}_\varepsilon^2 \mathbf{Z}_N \qquad (2)$$

$\mathbf{I}_N = \mathbf{I}_r \otimes \mathbf{I}_c \otimes \mathbf{I}_k$; $\mathbf{Z}_\delta = \mathbf{I}_r \otimes \mathbf{1}_c \otimes \mathbf{1}_k$ and $\mathbf{Z}_\gamma = \mathbf{I}_r \otimes \mathbf{1}_c \otimes \mathbf{1}_k$. (3)

Where $\mathbf{1}$ and $\mathbf{I}$ are as follows:

$$\mathbf{1}_r = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1_r \end{bmatrix}; \mathbf{1}_c = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1_c \end{bmatrix}; \mathbf{1}_k = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1_k \end{bmatrix} \text{ and} \qquad (4)$$

$$\mathbf{I}_r = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1_r \end{bmatrix}; \mathbf{I}_c = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1_c \end{bmatrix}$$

and $\mathbf{I}_k = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1_k \end{bmatrix}$ (5)

variance ratio, $\eta_\delta = \frac{\sigma_\delta^2}{\sigma_\varepsilon^2}$ and $\eta_\gamma = \frac{\sigma_\gamma^2}{\sigma_\varepsilon^2}$; introduced in matrix notation as

$$\mathbf{V} = \sigma_\varepsilon^2(\eta_\delta \mathbf{Z}_\delta \mathbf{Z}_\delta' + \eta_\gamma \mathbf{Z}_\gamma \mathbf{Z}_\gamma' + \mathbf{I}_N) \quad (6)$$

The general mixed model Y, as equation (1), contains its generalized least-square (GLS) estimator β as

$$\hat{\boldsymbol{\beta}}_{\mathrm{GLS}} = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y} \quad (7)$$

with its variance-covariance matrix

$$\mathrm{cov}(\hat{\boldsymbol{\beta}}) = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1} \quad (8)$$

3.3 D-optimal criteria

Strip-Strip-plot structure produces a large number of runs. In order to reduce unnecessary runs, D-optimal criteria is introduced. The criteria utilizes the determinant of design information matrix $(\mathrm{M} = \mathrm{X'V^{-1}X})$ to determine the number of runs. The design efficiency, Deff, is as follows (Borkowski, 2015):

$$\mathrm{D_{eff}} = 100\left(\frac{|\mathbf{X'V^{-1}X}|^{1/p}}{\mathbf{N}}\right) \quad (9)$$

p is the number of parameters and N is total runs.

Balancing and orthogonal properties are the key characteristics that will determine the quality of any design. The more of these properties exist, the more efficiency in the designs.

3.4 Factorial design within strip-strip-plot structure

For stamping process, the fitted model is accompanied by first order multiple regression model, thus the strip-strip-plot model for three-stage processes is as follows:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}_\delta\boldsymbol{\delta} + \mathbf{Z}_\gamma\boldsymbol{\gamma} + \boldsymbol{\varepsilon} \quad (10)$$

The expansion of the model with six factors will be

$\mathbf{X}\boldsymbol{\beta} = \beta_0$

$+\boldsymbol{\beta}_1 x_1 + \boldsymbol{\beta}_2 x_2 + \boldsymbol{\beta}_{12} x_1 x_2$ ... stage-1 effects

$+\boldsymbol{\beta}_3 x_3 + \boldsymbol{\beta}_4 x_4 + \boldsymbol{\beta}_{34} x_3 x_4$ ... stage-2 effects

$+\boldsymbol{\beta}_5 x_5 + \boldsymbol{\beta}_6 x_6 + \boldsymbol{\beta}_{56} x_5 x_6$ ... stage-1 effects

$+\boldsymbol{\beta}_{13} x_1 x_3 + \boldsymbol{\beta}_{14} x_1 x_4 + \boldsymbol{\beta}_{23} x_2 x_3 + \boldsymbol{\beta}_{24} x_2 x_4$

... stage-1xstage-2 effects

$+\boldsymbol{\beta}_{15} x_1 x_5 + \boldsymbol{\beta}_{16} x_1 x_6 + \boldsymbol{\beta}_{25} x_2 x_5 + \boldsymbol{\beta}_{26} x_2 x_6$

... stage-1xstage-3 effects

$+\boldsymbol{\beta}_{35} x_3 x_5 + \boldsymbol{\beta}_{36} x_3 x_6 + \boldsymbol{\beta}_{45} x_4 x_5 + \boldsymbol{\beta}_{46} x_4 x_6$

... stage-2xstage-3 effects (11)

The predicted model $\hat{\mathrm{Y}}$ contains all stage effects of main, within-stage interaction and cross-stage interaction effects.

4. Experiment Methods

4.1 Constructing experimental design

4.1.1 Chosen processes, factors and level

Three critical processes, that consist of forming (stage-1), flanging (stage-2) and piercing2 (stage-3), respectively, are chosen to perform three multi-stage experiment with full factorial design for all stages. All stage factors are assigned with two levels of -1, low, and 1, high. They are defined as A and B for die height and slide speed of forming process, C and D for die height and slide speed of flanging process and E and F for die height and slide speed of piercing2, respectively.

Table 3 Full factorial strip-strip-plot design with 64 runs, D_{eff} = 100%

		Flanging			
		C D -1 -1	C D -1 1	C D 1 -1	C D 1 1
		Piercing2	Piercing2	Piercing2	Piercing2
Forming	A B	E F	E F	E F	E F
	-1 -1	-1 -1	-1 -1	-1 -1	-1 -1
		-1 1	-1 1	-1 1	-1 1
		1 -1	1 -1	1 -1	1 -1
		1 1	1 1	1 1	1 1
	-1 1	-1 -1	-1 -1	-1 -1	-1 -1
		-1 1	-1 1	-1 1	-1 1
		1 -1	1 -1	1 -1	1 -1
		1 1	1 1	1 1	1 1
	1 -1	-1 -1	-1 -1	-1 -1	-1 -1
		-1 1	-1 1	-1 1	-1 1
		1 -1	1 -1	1 -1	1 -1
		1 1	1 1	1 1	1 1
	1 1	-1 -1	-1 -1	-1 -1	-1 -1
		-1 1	-1 1	-1 1	-1 1
		1 -1	1 -1	1 -1	1 -1
		1 1	1 1	1 1	1 1

4.1.2 Balanced data pattern of factorial design within strip-strip-plot structure

For balanced data, cell or stage-3 will contain equal number of runs. The symmetry of data structure will provide balancing and orthogonal properties design which its information matrix M is high enough compared to our acceptance level.

In the case of utilizing full factorial design, each stage contains $2^2 = 4$ runs. Then, for all three chosen stages, all combinations are $4 \times 4 \times 4 = 64$ runs. The design will contain fully balancing and orthogonal properties with design efficiency Deff is 100%. The cross pattern of all sixty-four runs are shown in Table 3.

Unfortunately, the sixty-four runs acquire high experimental cost, thus unnecessary runs have to be eliminated. According to observation, all cells, or stage-3, contain the same four treatments. So, we will reduce some treatments in stage-3 only. This means that the balancing and orthogonal properties still remain in both stage-1 and stage-2.

Table 4 Full factorial strip-strip-plot design with 32 runs, $\eta_\delta = \eta_\gamma = \boldsymbol{\sigma}_\varepsilon^2 = 1$ and $D_{eff,}$ = 80%

		Flanging			
		C D -1 -1	C D -1 1	C D 1 -1	C D 1 1
		Piercing2	Piercing2	Piercing2	Piercing2
Forming	A B	E F	E F	E F	E F
	-1 -1	-1 -1 1 1	-1 1 1 -1	-1 1 1 -1	1 1 1 -1
	-1 1	-1 -1 1 1	-1 1 1 -1	-1 1 1 -1	-1 -1 1 1
	1 -1	-1 1 1 -1	-1 1 1 -1	-1 -1 1 1	-1 -1 1 1
	1 1	-1 1 1 -1	-1 1 1 -1	-1 -1 1 1	-1 -1 1 1

For stage-3, the balancing property still remains but the orthogonal is slightly reduced in accordance with accepted criteria D_{eff}. D-optimal searching algorithm is utilized to randomly extract from some unnecessary runs until reaching D_{eff}, > 70%. Each cell in stage-3 still contain the same number of runs. With the algorithm, we are able to generate pattern which the total runs are reduced from 64 to 32 runs. Together with using the variance ratios η_δ, η_γ and error $\boldsymbol{\sigma}_\varepsilon^2$ are all equal to 1.0 resulting in 80% D-efficiency. The design structure is shown in Table 4.

Table 5 Full factorial strip-strip-plot design with 24 runs, $\eta_\delta = \eta_\gamma = \boldsymbol{\sigma}_\varepsilon^2 = 1$ and D_{eff}, = 69.87%

		Flanging	Flanging	Flanging	Flanging
		C D -1 -1	C D -1 1	C D 1 -1	C D 1 1
		Piercing2	Piercing2	Piercing2	Piercing2
Forming	A B	E F	E F	E F	E F
Forming	-1 -1		-1 1 1 -1	-1 1 1 -1	1 1 1 -1
Forming	-1 1	-1 -1 1 1		-1 1 1 -1	-1 -1 1 1
Forming	1 -1	-1 1 1 -1	-1 1 1 -1		-1 -1 1 1
Forming	1 1	-1 1 1 -1	-1 1 1 -1	-1 -1 1 1	

4.1.3 Unbalanced data pattern of factorial design within strip-strip-plot structure

In this case, the further reduction of unnecessary runs continue in stage-3 only resulting to some empty cells as shown in Table 5.

All main effects still contain both balancing and orthogonal properties. However, there are quite a few two-way effects that are missed in both balancing and orthogonal properties. With the D-optimal algorithm, we are able to generate pattern in which the total runs are reduced from 32 to 24 runs. With utilizing the variance ratios η_δ, η_γ and error σ_ε^2 equaled to 1.0, resulting in D-efficiency is 69.87%.

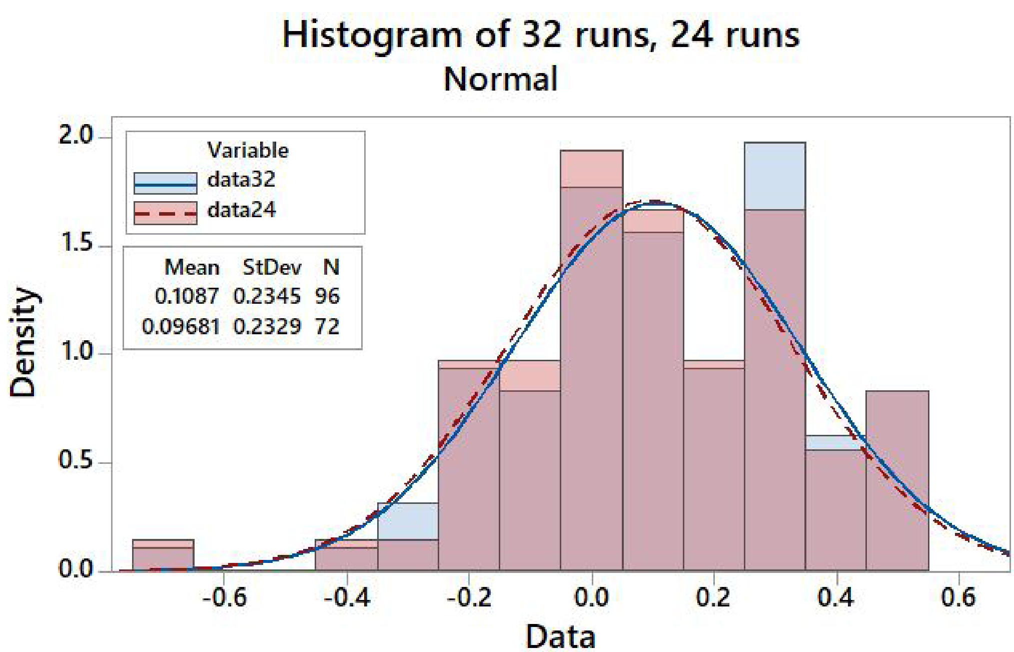
4.2 Process and equipment

The real experiment is performed at the stamping factory, Sangcharoen Tools Center Co., Ltd., where we can utilize five stamping sub-processes, real metal sheets and production people as follows:

- Stamping production line- blanking, forming, flanging, piercing-1 and piercing-2 processes and machines.
- 110 pieces of battery plate carrier.
- Dimension measurement gauges and veneer calipers.
- 5 production operators and 1 measurement inspector.

5. Multiple Regression Analysis

Thirty-two runs of balanced data design and twenty-four runs of unbalanced data design with three replicates are experimented on stamping processes. The quality characteristic Y, the difference of left and right displacements, are measured. The quality of data is shown in Figure 2.



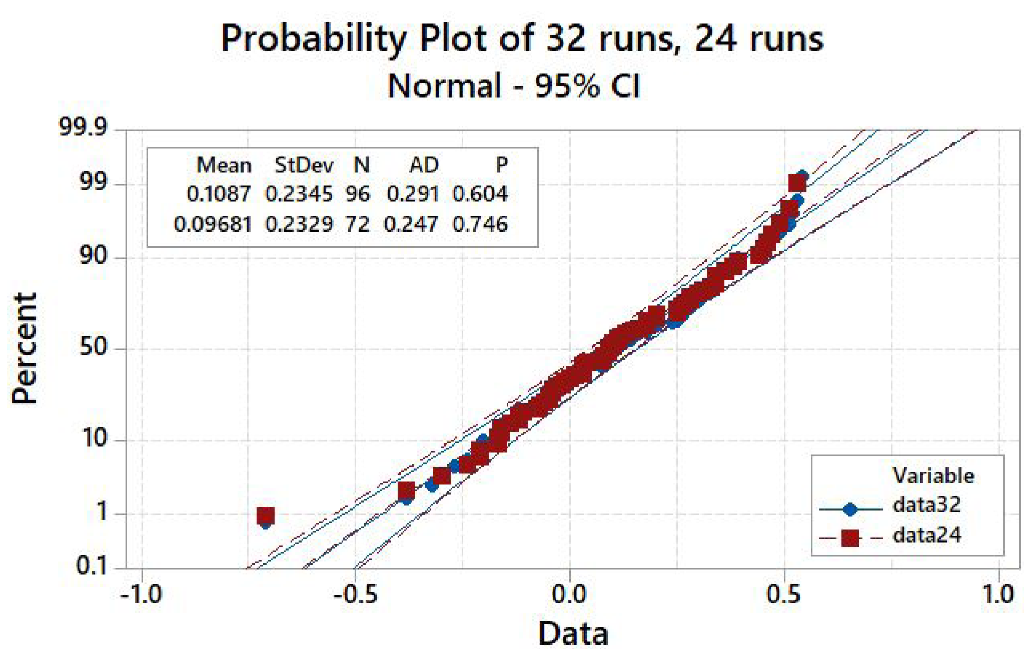


Figure 2 Histograms and probability plots of experimental data, 32 runs and 24 runs.

The probability plots at 95% confident level of balanced data and unbalanced data designs contain p-values of 0.604 and 0.746 that are higher than critical p-value of 0.05. These mean that both experiments provide data which are in the same normal distributions. In accordance with histograms, both of their means, 0.1087 mm. and 0.0968 mm. respectively, and standard deviations, 0.2345 mm. and 0.2309 mm. respectively, are slightly different resulted that experimental data contain both properties of accuracy and precision.

Model estimation $\hat{\beta}$ of multiple regression analysis, is obtained from equation (7), $\hat{\beta}_{GLS} = (X'V^{-1}X)^{-1}X'V^{-1}Y$. According to balanced data pattern of 32 runs, the generalize estimator $\hat{\beta}_{GLS}$ is equivalent to the ordinary estimator $\hat{\beta}_{OLS}$. Then we can utilize standard software, such as minitab 17, to perform computation in which its result is shown in Table 6.

Table 6 $\hat{\beta}$ Model estimation of balanced data design 32 runs

$\hat{\beta}$ Model Estimation

Stage	Variable	Regression Coefficient	P-value
	Const	0.1088	0.0000
1	A	-0.0117	0.6223
1	B	-0.0063	0.7917
2	C	-0.0275	0.3157
2	D	-0.0002	0.9930
3	E	-0.0541	0.0648
3	F	-0.0121	0.6099
2x3	CF	0.0798	0.0190
2x3	DE	-0.0605	0.0392
3	EF	0.0567	0.0405

ANOVA table

Effects	df	MS	F	P-value
Main	6	0.0454	0.85	0.536
Two-way	3	0.1926	3.6	0.017
Errors	86	0.0535		
Total	95			

There are two cross-stage interaction effects that are statistically significant with p-value equal to 0.05. CF, flanging die height interacted with piercing-2 slide speed, whereas DE, flanging slide speed interacted with piercing-2 die height. This means that both flanging parameters provide impact to the next stage piercing-2 factors. Another interaction effect is within-stage interaction effect EF, piercing-2 die height interacted with piercing-2 slide speed. These significant cross-stages effects will be included into predicted model which will provide more accuracy into its optimized model.

For unbalanced data pattern of 24 runs, the generalized estimator, $\hat{\beta}_{GLS} = (X'V^{-1}X)^{-1}X'V^{-1}Y$, is performed specially with coding in SAS software or manual calculation in excel worksheet. Our calculation of the 24-runs GLS estimators, compared with 32-runs estimators, are shown in Table 7.

Table 7 $\hat{\beta}$ Model estimators of balanced data 32-runs and of unbalanced data 24 runs

Variables	$\hat{\beta}$ of balanced 32-runs	$\hat{\beta}$ of unbalanced 24-runs
Const	0.1088	0.1027
A	-0.0117	-0.0310
B	-0.0063	-0.0100
C	-0.0275	-0.0533
D	-0.0002	-0.0040
E	-0.0541	-0.0650
F	-0.0121	-0.0317
CF	0.0798	0.0918
DE	-0.0605	-0.0125
EF	0.0567	0.0696

Both of the estimators are slightly different in magnitudes, thus their predicted models $\hat{Y}$'s will provide values which are slightly different too. For our target, the different displacements, should be zero. Five optimized runs are predicted by these two predicted models with their predicted displacements, $\hat{Y}$'s, as shown in Table 8.

Table 8 Optimized runs with target = 0 and their predicted values

Parameters	Run1	Run2	Run3	Run4	Run5
A	1	0	0	0.2	0.5
B	1	0	0	0	0.5
C	-1	0	1	1	0.5
D	-1	0	0	0	0.5
E	1	0	0.5	0.5	0.5
F	-1	0	0	0	0.5
$\hat{Y}$-32	0.1601	0.1088	0.0543	0.0519	0.0719
$\hat{Y}$-24	0.1204	0.1027	0.0169	0.0107	0.0424

Two-Sample T-Test and CI: y32, y24

```
Two-sample T for y32 vs y24

     N    Mean   StDev  SE Mean
y32  5  0.0894  0.0456    0.020
y24  5  0.0586  0.0501    0.022


Difference = μ (y32) - μ (y24)
Estimate for difference:  0.0308
95% CI for difference:  (-0.0409, 0.1024)

T-Test of difference = 0 (vs ≠):
T-Value = 1.01  P-Value = 0.344  DF = 7
```

Figure 3 Hypothesis t-test between predicted values $\hat{Y}$ of balanced data 32 runs and unbalanced data, 24 runs

Both optimized runs from their models are able to get closed to zero value. However the predicted values from unbalanced data design (24-runs) are slightly closer to zero than those of balanced data design (32-runs). Thus, a hypothesis t-test is performed and its result shows that there are no statistical difference in estimators from both designs. The t-test result from minitab 17 is shown in Figure 3 in which p-value is 0.344.

Conclusion

In this paper, we discuss factorial design within strip-strip-plot structure for three multi-stage stamping processes in order to reduce accumulated variations at final stage. Two designs of balanced data with 32 runs and unbalanced data with 24 runs are generated with the D-optimal criteria of design efficiency, Deff, at 80% and 69.87% respectively. The predicted models of both designs are able to extract all stage' effects of main, within-stage interaction and cross-stage interaction effects significantly.

Five optimized runs with their target, equaled zero, are predicted from both predicted models. Both of their means are evaluated by hypothesis t-test which interprets that there are no significant difference. This means that, we can confirm experimental runs with unbalanced data design of 24 runs in which some experimental cost is saved 25% than that of 32 runs.

Even though unbalanced data design can provide us minimal runs with acceptable efficiency, such as D-optimal criteria, its mathematical difficulty is the obstacle to widely utilization. Furthermore, there are many efficiency criteria that are still under researched, such as D-optimal, A-optimal, G-optimal, etc.

Recommendation

Unbalanced data designs with the optimal criteria will benefit to optimize number of experimental runs for multi-stages DOE which takes advantages both analysis model and minimal number of runs than those of single DOE. To get multi-stages DOE patterns and generalized estimators, one has to create them in excel worksheet or SAS software coding. Further researches will benefit in developing unbalanced data design patterns in different kinds of designs, such as mixture design, extending number of stages and software developing for generalized estimator calculation.

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